CAIE Physics A-level

Topic 23: Nuclear Physics

Notes

23 - Nuclear Physics

23.1 - Mass Defect and Nuclear Binding Energy

Energy and mass are related through an equation known as the **mass-energy equivalence** proposed by Albert Einstein in 1905. This famous equation is written as

$$E = mc^2$$

where $c = 3 \times 10^8\ ms^{-1}$ (the speed of light), E is energy in joules and m is mass in kilograms. Hence a kilo of matter is equivalent to $9 \times 10^{16}\ J$ of energy.

The joule is too large a quantity to be used conveniently in the realm of nuclear physics, so it's usually replaced with the electronvolt described in the previous section. Since nuclear interactions are more energetic than the electron interactions from the previous section, we will use the mega-electronvolt (MeV) which is equivalent to one million electronvolts.
The conversion from Joules to MeV is therefore $1\ MeV = 1.60 \times 10^{-13}\ J$.
Similarly, instead of using kilograms, we can also measure the mass in terms of the more appropriate **atomic mass unit u**, where $1\ u = 1.66 \times 10^{-27}\ kg$.
Converting between the two would yield the relation $1\ u = 931\ MeV$.

Using these relations we can discover a property of nuclei known as the **mass defect**. The masses in atomic units of the proton, neutron, and electron are given below to 6 dp:
Proton mass, m_p = 1.007276 u
Neutron mass, m_n = 1.008665 u
Electron mass, m_e = 0.000549 u

By combining these values, we could estimate the mass of a helium-4 nucleus as
$(2 \times 1.007276\ u) + (2 \times 1.008665\ u) = 4.031882\ u$.
However, if we measure the actual mass of a helium-4 nucleus, we would get 4.001508 u, which gives us a **mass defect** of 0.030374 u.

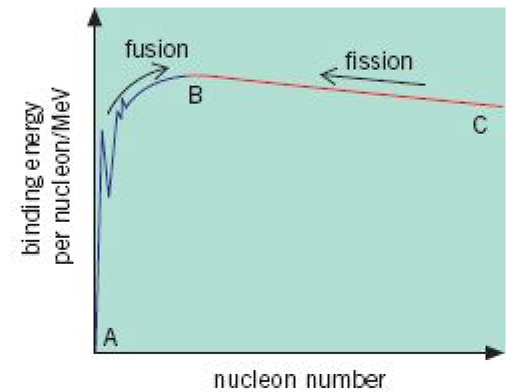
The mass defect is defined as the difference between the actual measured mass of the nucleus and the summed mass of its constituent particles.

In the nucleus, the **strong nuclear force** binds the nucleons together. Separating these completely requires energy, this is known as the **binding energy**. The more stable the nucleus is, the higher its binding energy will be. Binding energy is effectively equivalent to the mass defect.

$${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$$

Nuclear equations can be written in the form on the right. The superscript number before each element letter is the atomic mass (nucleon number), while the lower number is the atomic number (proton number). The total atomic mass and total atomic number respectively - as well as the total mass-energy of the system - must be conserved in any interaction. Checking that these conditions are met is important when writing nuclear equations.

In order to measure the stability of a nucleus, we can consider the **binding energy per nucleon**. This is simply the binding energy of each nucleus divided by its number of nucleons. The higher this quantity is the more stable the nucleus will be.
Plotting the relationship of binding energy per nucleon against the nucleon number gives a curve peaking at the iron-52 nucleus (labelled as B on the diagram).



From hydrogen-1 (A) up to the peak at iron-52, the binding energy per nucleon generally increases, with some peaks and troughs on the way. All the nuclei heavier than iron-52 have lower binding energies per nucleon, so they are less stable. Iron-52 is therefore the most stable nucleus.

For nuclei from A-B, it is energetically favourable for smaller nuclei to combine and form a larger nucleus with a higher binding energy per nucleon. This process is called **nuclear fusion**. For nuclei from B-C, it is instead more favourable to split the larger nuclei apart into smaller ones with a higher binding energy per nucleon. This process is called **nuclear fission**. The energy released by these nuclear reactions is given by the mass-energy equivalence relation $E = \Delta mc^2$ where Δm is the mass defect between the reactants and the products of the nuclear reaction.

23.2 - Radioactive Decay

Unstable nuclei will undergo a process called **radioactive decay** where they will emit particles or radiation in order to become more stable. Radioactive decay can be observed by detecting what is known as the **count rate**, and this shows us that the decay is both **spontaneous and random**.

Radioactive decay is **spontaneous** because it is not influenced by external factors such as temperature or pressure, it depends only on the internal properties of the nucleus.

Radioactive decay is **random** because it cannot be predicted which nucleus in a given sample is going to decay and when. Instead, there is a fixed probability per unit time - the **decay constant** λ - that a nucleus will decay. The random nature of the decay appears from the count rate by the fact that the actual number of undecayed nuclei doesn't follow the mathematical description exactly, but rather fluctuates around it.

The **activity A** of a nucleus is related to the decay constant via the equation $A = \lambda N$ where N is the number of undecayed nuclei. This relation can be derived by considering the following steps:

If we define that there are N nuclei in a radioactive sample at a time t, then after an additional length of time dt, the sample will have decayed by dN nuclei such that at time (t+dt) there are (N-dN) nuclei remaining.
The probability of decay is given by $-\frac{dN}{N}$ while the decay constant is the probability per unit time, a negative sign is included because the decay constant is always positive and N decreases as t increases i.e.

$$\lambda = -\frac{dN}{N\,dt}$$

therefore

$$\frac{dN}{dt} = -A = -\lambda N$$

The activity is measured in becquerels (Bq). 1 Bq is equivalent to one decay per second.

The derivation of the solution to the equation $\frac{dN}{dt} = -\lambda N$ is slightly beyond the scope of the physics course, though knowledge of the solution itself is required. The extension steps are coloured in blue. The solution is obtained as follows:

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda\, dt$$

$$\int \frac{1}{N} dN = \int -\lambda\, dt$$

$$ln(N) = -\lambda t + C$$

where C is a constant of integration. Taking the exponential of both sides...

$$N = e^{-\lambda t}\, e^{C}$$

the solution is therefore

$$N = N_0 e^{-\lambda t}$$

where N_0 is the initial number of nuclei. A similar equation can be obtained for the activity based on the fact that $A = \lambda N \Rightarrow A \propto N$:

$$A = A_0 e^{-\lambda t}$$

The **half-life** of a radioactive sample is the time taken for half of the total nuclei in a sample to decay. Because activity is proportional to the number of nuclei, this is a constant property of any radioactive sample: it always takes the same length of time for half of a sample to decay.

The half-life $t_{1/2}\,(or\ T_{1/2})$ can be derived by finding the time at which the current nuclei population is half the initial population:

$$N = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$
$$\frac{1}{2} = e^{-\lambda t}$$
$$2 = e^{\lambda t}$$

therefore

$$t_{1/2} = \frac{ln2}{\lambda} \approx \frac{0.693}{\lambda}$$

The change in N over time is shown in the graph below. The constant half life is marked on the axes.